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Banking Competition, Production Externalities, and the Effects of Monetary Policy

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Abstract

Since the global financial crisis, there has been a significant amount of concern about the presence of large-scale financial intermediaries which affects the competitive landscape of the banking sector in advanced economies. In light of this issue, this paper develops a framework to demonstrate how the degree of concentration impacts capital accumulation and economic growth. As is standard in the growth literature, we incorporate production externalities from the aggregate capital stock which promote economic development and growth over time. Notably, the model demonstrates that higher degrees of concentration distort economic activity by interrupting the externalities from capital investment. We also show that the ability of monetary policy to provide a favorable growth environment and achieve price stability is hampered by higher degrees of concentration. Consequently, the task of central banking will be more difficult as the sector further consolidates over time. Furthermore, the developing world will not be immune to these challenges facing regulators and policymakers in advanced economies.

JEL Codes: O42, D42, E52, G21

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1 Introduction

In recent years, there has been increasing concern about developments in the banking sector in advanced economies. During the global financial crisis of 2008, the terminology "Too Big to Fail" became common parlance. Yet, the increased consolidation was not an isolated event – it has been a long-term development that has been taking place over many years. For example, in 1989, there were nearly 19,000 different financial institutions that were active in the United States. Nearly a decade later, the number was reduced to about half. Similar developments have occurred in other advanced economies. Eventually,

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the process could spillover to the developing world so that the global financial system will become even more concentrated than today.

What are the consequences of banking sector concentration for economic growth and development over time? Since the initial work by King and Levine (1993), an abundance of research has shown that the level of development of the banking system plays an important role in promoting equipment investment and growth. Moreover, the empirical literature has been complemented by theoretical models that articulate how the functions of financial intermediaries foster higher rates of progress. In particular, Bencivenga and Smith (1991) demonstrate that an active intermediary sector encourages risk-sharing and "eliminates excessive investment in unproductive liquid assets... [where the absence leads towards] a composition of savings that is unfavorable to capital accumulation." Consequently, it would be naive to believe that the growth process is independent from the industrial organization of the banking sector.

We view that the increasing concentration of financial intermediaries presents a distortion that is likely to impede capital investment and lead to lower economic growth. The reason is that banks with more market power do not provide the same functions that support investment in productive assets as perfectly competitive banks. In fact, they have a tendency to hoard cash reserves as they behave strategically. This important centerpiece of our analysis coincides with arguments by Bencivenga and Smith (1991). Furthermore, Ghossoub and Reed (2014, 2015) document that banking systems in concentrated sectors allocate more resources toward liquid assets. In addition, a number of recent papers have shown that banking concentration generally depresses economic growth across countries.[1]

Moreover, based upon the pioneering work of Romer (1986), the decrease in capital formation is troublesome because externalities from capital formation have been shown to play an important role in the process of economic growth and development. In particular, in his framework, the size of the aggregate capital stock raises productivity and contributes to increasing returns to scale across the economy. Subsequent work has sought to examine whether such external economies are empirically relevant. For example, in two influential papers, De Long and Summers (1991, 1993) contend that equipment investment is associated with large production externalities. In fact, De Long and Summers (1993) argue that the social return to equipment investment can be as high as 30% in advanced economies. In related work, Jones (1994) stresses that the tax treatment of capital goods is critically important for growth and welfare. Easterly (1993) also emphasizes that distortions in markets can impede capital accumulation and thereby lead to lower economic growth.

In order to study the impact of financial sector competition for economic growth and development, we follow Bencivenga and Smith (1991) by studying an overlapping generations version of the classic Diamond and Dybvig (1983) model

[1] In particular, see Cetorelli and Gambera (2001) and Deidda and Fattouh (2005). By comparison, Claessens and Laeven (2005) do not generally find evidence that banking sector competition is related to growth. However, they do observe that more competition promotes growth in industries that are more dependent on external financing.

with production. As in Schreft and Smith (1997, 1998), limited communication leads to an important transactions role for money. Banks can invest in both physical capital and money balances. However, in contrast to the standard Schreft and Smith framework, we follow Ghossoub and Reed (2014, 2015) where imperfectly competitive financial intermediaries behave strategically in markets. Consequently, the degree of concentration in the banking sector has a significant impact on risk-pooling and investment activity.[2]

Based upon the insights and contributions since Romer (1986), our model is distinct from the previous work as we incorporate externalities from capital formation as in Bhattacharya et al (2009). Yet, in contrast, financial intermediaries act as Courtnot-competitors in capital markets.[3] Notably, following the insights of Easterly, increasing concentration interrupts the externalities from capital formation and acts as a drag on growth. It is our objective to demonstrate that the ability of monetary policy to foster a favorable growth environment also depends on the degree of concentration in the sector.[4]

Though our ultimate goal is to study the implications of banking concentration for economic growth, a wide array of work in the monetary growth literature follows the neoclassical growth or 'exogenous' growth framework. Therefore, our analysis begins in Section 2 by studying a setting in which there are externalities from physical capital but diminishing returns to capital accumulation apply. As in Ghossoub and Reed (2014), an increase in the degree of concentration leads to lower capital formation and higher nominal returns to capital, but better risk-sharing in Section 3. A standard Tobin asset substitution channel is at work where money growth leads to a higher amount of capital formation. The efficacy of monetary policy to promote capital formation depends on the degree of concentration in a non-linear way. If the banking sector is initially relatively competitive, some consolidation renders monetary policy to be more effective in stimulating investment because institutions will hold more money balances so that the asset substitution channel is at work. However, if the size distribution favors a small number of large intermediaries, further consolidation weakens the ability of policy to promote economic activity.

In contrast to Ghossoub and Reed (2014), we also incorporate externalities from capital formation. As suggested by De Long and Summers (1991, 1993), externalities from the capital stock in our model promote capital formation and income. However, they lead to weaker risk-sharing. The interesting part comes from the interactions with monetary policy.

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[2] In terms of credit markets, see Hannan (1991) and Corvoisier and Gropp (2002) who observe that interest rates on loans depend on concentration ratios in markets. In addition, Beck, Demirguc-Kunt and Maksimovic (2003) point out that concentrated banking systems are associated with more credit rationing.

[3] Cetorelli and Peretto (2010) study a neoclassical growth framework with Cournot competition in capital markets but there is not a role for money so they do not consider the connections between the efficacy of monetary policy and the degree of competition.

[4] Both Cechetti (1999) and Kashyap and Stein (1997) study how the effectiveness of monetary policy depends on competition in the banking system. Notably, Cechetti observes that the impact of policy shocks on output is higher if the banking sector is more competitive. See also the empirical evidence in Ghossoub and Reed (2014, 2015).

As one would expect, monetary policy plays a larger role in capital formation in the presence of stronger externalities. However, the ability to do so critically depends on the degree of banking sector concentration. Consequently, the impact of central bank actions on investment and aggregate income revolves around two sources of market imperfections – externalities from capital formation and strategic behavior by imperfectly competitive financial intermediaries.

Notably, in Section 4, we find that the welfare-maximizing money growth rule reflects these dual externalities. First, the welfare-maximizing money growth rule is the minimal growth rate in which a monetary steady-state exists if externalities from physical capital are low. However, over some intermediate range of the strength of the external effects, it is optimal to deviate from such a rate in order to promote capital formation. Moreover, the optimal money growth rate is higher if the sector is more concentrated. Therefore, in economies where both mechanisms are at work – large external effects and high concentration – central banks can play a critical role in providing an environment that fosters capital accumulation and high levels of welfare.

Finally, we turn to our ultimate objective – to study the implications of concentration for economic growth. Thus, we extend the model to incorporate perpetual economic growth in Section 5. We show that real variables grow at a common growth rate along the balanced growth path. In addition, the nominal return to capital is stationary. The conditions for existence also depend on the size of the banking sector. As previously suggested, we find that higher degrees of concentration are associated with lower growth. Therefore, policymakers should be concerned about the consolidation trend that has developed in advanced economies. If they continue to experience slower rates of progress, it will have important implications for the global economy.

In contrast to the economy in the absence of growth, the inflation rate is an endogenous outcome that is not completely pinned down by the rate of money growth. Interestingly, the inflation rate also depends on the degree of concentration – sectors with greater concentration tend to have higher inflation since they hold more money balances. However, analogous to Section 3, the ability of monetary policy to stimulate economic growth depends on the degree of concentration in non-linear ways.

Moreover, central banks over time have come to adopt a greater focus on price stability. Our model offers some important insights into the ability to do so – if the sector is more concentrated, the influence of monetary policy on the inflation rate will be weaker.

Consequently – regardless of a central bank's mandate – economic growth or explicit inflation targets – the consolidation of the financial sector poses numerous challenges. Therefore, the long-term trends towards further consolidation should be a central concern for banking authorities across countries. And, the developing world will not be immune to the repercussions from the financial sectors of advanced economies.

2 Environment

Consider a discrete time economy that is divided into two geographically separated, yet symmetric locations. Let time be indexed by $t = 1, 2, ..\infty$. At the beginning of each period, a unit mass of two-period lived workers and $N > 1$ bankers are born. Let each bank be indexed by j, with $j = 1, 2, ...N$. Each young worker is endowed with one unit of labor effort which she supplies inelastically and is retired when old. Moreover, workers are risk averse and value only their old age consumption, c_{t+1}. The preferences of a typical worker are such that $u(c_t) = \frac{c_t^{1-\theta}}{1-\theta}$ where $\theta \in (0, 1)$ is the coefficient of risk aversion. By comparison to workers, bankers do not receive any endowments, are risk neutral, and only value their old age consumption.

In each location, the single and perishable consumption good is produced by a representative firm using capital and labor as inputs. The firm has access to a production technology of the form $Y_t = A\overline{K}_t^{\rho} K_t^{\alpha} L_t^{1-\alpha}$ where Y_t, K_t, and L_t are the firm's period t output, capital stock, and labor, respectively. The average capital stock $\overline{K}_t$ is taken by the firm as given and provides positive externalities. The parameter $\rho \in [0, 1-\alpha)$, reflects the strength of the investment externality. For example, a higher value of ρ implies more pronounced spillovers from the economy's aggregate level of investment into individual production. However, in contrast to Romer (1986), the externality from physical capital is not strong enough to generate perpetual growth.[5] Therefore, diminishing returns to capital still apply at the individual firm and aggregate levels. In addition, $\alpha \epsilon (0, 1)$ is capital's share of total output and A reflects total factor productivity. Further, the capital stock completely depreciates in the production process.

There are two assets in the economy, physical capital and fiat money. One unit of goods allocated towards investment in physical capital in period t becomes one unit of capital in period $t+1$. In addition to physical capital, there is a stock of money (fiat currency) that circulates in the economy. We denote the per worker nominal monetary base as M_t. Money is a universally recognizable, durable, and divisible object. At the initial date 0, the generation of old workers at each location is endowed with the aggregate stock of capital (K_0) and money supply (M_0). Since the population of workers is equal to one, these variables also represent aggregate values. Assuming that the price level is common across locations, P_t is the number of units of currency per unit of goods at time t.

Following Schreft and Smith (1997, 1998), workers are exposed to idiosyncratic risk. To be specific, there is a positive probability that they will need to conduct transactions in the opposite location. As is standard in Schreft-Smith type models, the transaction location shock is viewed as a "random relocation" shock which occurs with probability π. In particular, the realization of the relocation shock will not occur until after financial portfolio allocations have been implemented. As the number of workers is unity, the probability of relocation also reflects the number of agents that will be relocated (movers) on each island. Though the total number of agents exposed to the relocation shock is

[5] Clearly, the standard AK model is obtained when $\rho = 1 - \alpha$.

public information, the individual's realization is privately observed.

Furthermore, there is limited communication between different locations. That is, on the opposite island, workers are viewed as "anonymous" individuals. Therefore, agents cannot trade claims to assets they own in their original location. However, fiat money can be used to overcome these frictions. Moreover, it is the only asset that can be moved across islands. As a result, workers who learn they will be relocated will liquidate all their asset holdings into currency. In this manner, financial intermediaries (bankers) play an important role in terms of insuring workers against the idiosyncratic risk. This is similar to Diamond and Dybvig (1983) with the exception that there is a finite number of intermediaries and we study a production economy in which bankers exploit their market power. In contrast to workers, bankers are not subject to relocation shocks.

The final agent in this economy is a government (or central bank) that adopts a constant money growth rule. Denote the real aggregate money stock in period t by $\tilde{m}_t$. The evolution of real money balances between periods $t-1$ and t is expressed as:

$$\tilde{m}_t = \sigma \frac{P_{t-1}}{P_t} \tilde{m}_{t-1} \tag{1}$$

where $\sigma > 0$ is the gross rate of money creation (or destruction when $\sigma < 1$) chosen at the beginning of time and $\frac{P_{t-1}}{P_t}$ is the gross rate of return on money balances between period $t-1$ and t. The government rebates seigniorage income to young workers through lump-sum transfers. Denote the total amount of transfers at the beginning of period t by τ_t, where

$$\tau_t = \frac{\sigma - 1}{\sigma} \tilde{m}_t \tag{2}$$

3 Trade

3.1 Factors Markets

In period t, a representative firm rents capital and hires workers in perfectly competitive factor markets at rates r_t and w_t, respectively. The inverse demands for labor and capital by a typical firm are:

$$w_t = (1-\alpha) A \overline{K}_t^{\rho} K_t^{\alpha} L_t^{-\alpha} \tag{3}$$

and

$$r_t = \alpha A \overline{K}_t^{\rho} K_t^{\alpha-1} L_t^{1-\alpha} \tag{4}$$

3.2 A Typical Worker

A young worker born in period t works and earns the wage rate (w_t) and receives τ_t units of goods in real injections from the government. As she only values old age consumption the worker saves her entire income. Moreover, as agents are subject to relocation shocks, all savings are intermediated.

3.3 A Typical Bank's Problem

In this environment all bankers are identical and solve the same problem. Therefore, we omit the indexation for each bank. Moreover, banks compete over prices in the deposit market. In particular, at the beginning of period t, each bank announces deposit rates taking the announced rates of return of other banks as given. For every unit of deposits, each depositor is promised a gross real return of r_t^m if she relocates and r_t^n if not. As financial services provided by banks to their depositors are identical, each financial institution receives the same amount of deposits equal to $\frac{w_t+\tau_t}{N}$ from $1/N$ depositors. The deposits received by a bank are allocated towards cash reserves and capital goods. Let m_t and k_{t+1} respectively denote the real amount of cash balances and capital goods held by each bank.

In contrast to the deposit market, the rental market is characterized by Cournot (quantity) competition. That is, each bank recognizes that its own decisions about the amount of capital supplied will affect the market rental rate, but that its choice does not affect that of other banks.

In equilibrium, price competition among banks for depositors will force them to choose return schedules and portfolio allocations to maximize the expected utility of a representative depositor. A bank's objective function is:

$$\underset{r_t^m, r_t^n, m_t, k_{t+1}}{Max} \frac{\pi \left[r_t^m \left(w_t+\tau_t\right)\right]^{1-\theta}+(1-\pi)\left[r_t^n \left(w_t+\tau_t\right)\right]^{1-\theta}}{1-\theta} \tag{5}$$

subject to the following set of constraints.

First, a bank's balance sheet at the beginning of period t is expressed by:

$$\frac{1}{N}\left(w_t+\tau_t\right)=m_t+k_{t+1} \tag{6}$$

Furthermore, payments to relocated agents must be made in cash:

$$\frac{\pi}{N} r_t^m \left(w_t+\tau_t\right)=m_t \frac{P_t}{P_{t+1}} \tag{7}$$

In this setting, we choose to study equilibria in which money is dominated in rate of return. Therefore, a banker will not hold excess reserves and its total payments to non-movers are paid out of its revenue from renting capital to firms in $t+1$:

$$\frac{1-\pi}{N} r_t^n \left(w_t+\tau_t\right)=r\left(k_{t+1}\right) k_{t+1} \tag{8}$$

In addition, the contract between the banks and its depositors has to be incentive compatible to prevent agents from lying about their types ex-post. That is:

$$r_t^m \leq r_t^n \tag{9}$$

Finally, as stated above, each bank faces the market's inverse demand for capital with:

$$r\left(k_{t+1}\right) = \alpha A K_{t+1}^{\rho+\alpha-1} L_{t+1}^{1-\alpha} \tag{10}$$

where $K_{t+1} = \overline{K}_{t+1} = \sum_{j=1}^{N} k_{t+1}^{j}$.

It is important to highlight here that the ability of banks to exert their market power on capital markets depends on the strength of the investment externality. For example, in the standard AK model, where $\rho = (1-\alpha)$, the demand for capital is perfectly elastic. However, as $\rho + \alpha < 1$, diminishing returns to capital investment apply so that banks face downward-sloping demand curves for capital.

The solution to the problem yields the demand for money balances by a single financial institution:

$$m_t = \frac{\frac{w_t + \tau_t}{N}}{1 + \frac{1-\pi}{\pi}\left[1 - \frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}}} \tag{11}$$

where $I_t = r_{t+1}\frac{P_{t+1}}{P_t}$ is the nominal return to capital between period t and $t+1$.

Equivalently, each bank allocates a fraction γ_t of its deposits towards cash reserves:

$$\gamma_t = \frac{1}{1 + \frac{1-\pi}{\pi}\left[1 - \frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}}} \tag{12}$$

Clearly, for a given level of deposits, the demand for money balances is strictly decreasing in the return to capital. Due to market power in the capital goods market, banks holds more cash reserves compared to a perfectly competitive outcome ($N \to \infty$). The term $\left[1 - \frac{(1-\alpha-\rho)}{N}\right] \in [0,1]$ represents the extent of market power and distortions that stem from imperfect competition. In particular, banks allocate a larger fraction of their deposits towards cash balances when market power increases (lower N). Finally, as the external effects from investment externality get stronger (higher ρ) the marginal return from capital is higher, which encourages banks to hold a less liquid portfolio.

Furthermore, substituting (6) and (11) into (7) and (8), the relative return to depositors is such that:

$$\frac{r_t^n}{r_t^m} = \left[1 - \frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1}{\theta}} \tag{13}$$

As banks hold more liquid portfolios when the banking system is more concentrated, they are able to provide their depositors with better insurance against liquidity shocks for a given return to capital. Moreover, depositors receive less insurance when the return to capital is higher.

From (13), the nominal return to capital must be sufficiently high for the self-selection constraint to hold. In particular, the incentive compatibility constraint holds when the nominal return is above some threshold level, $\underline{I} = \frac{1}{1-\frac{(1-\alpha-\rho)}{N}}$. Unlike standard random relocations models with perfect competition, the lower bound on the nominal return to capital is above zero. That is, a banking equilibrium is sustainable when

$$I_t \geq \underline{I} = \frac{1}{1-\frac{(1-\alpha-\rho)}{N}} > 1 \tag{14}$$

In this manner, the Friedman rule where money and capital yield the same real return cannot support a banking equilibrium.

3.4 General Equilibrium

In equilibrium, all markets will clear. In particular, labor receives its marginal product, (3), and the labor market clears, with $L_t = 1$. Furthermore, imposing that $K_t = \bar{K}_t = Nk_t$ on (3) and (4), the expression for wages and the real return to capital can be respectively expressed as:

$$w_t = (1-\alpha)\, AK_t^{\rho+\alpha} \tag{15}$$

and

$$r_t = \alpha AK_t^{\rho+\alpha-1} \tag{16}$$

Upon using (6), (11), the definition of τ_t, (2), and the expression for wages, (15) the law of motion of capital is such that:

$$K_{t+1} = \left(1 - \frac{1}{\frac{1-\pi}{\pi}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}}\sigma + 1}\right)(1-\alpha)\, AK_t^{\rho+\alpha} \tag{17}$$

In addition, from the evolution of money balances, (1), the inflation rate between t and $t+1$ is:

$$\frac{P_{t+1}}{P_t} = \sigma \frac{\tilde{m}_t}{\tilde{m}_{t+1}} \tag{18}$$

Furthermore, using the definition of I_t, (11), (15), and (16) in (18), equilibrium in the money market requires that the nominal return to capital evolves according to:

$$I_{t+1}=\frac{\left(\frac{I_t}{\alpha A\sigma}\left(\frac{1-\pi}{\pi}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}}I_t^{\frac{1-\theta}{\theta}}+\frac{1}{\sigma}\right)\frac{K_{t+1}}{K_t^{\rho+\alpha}}-\frac{1}{\sigma}\right)^{\frac{\theta}{1-\theta}}}{\left(\frac{1-\pi}{\pi}\right)^{\frac{\theta}{1-\theta}}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{1-\theta}}} \tag{19}$$

The loci characterized by (17) and (19) characterize the behavior of the economy at a particular point in time. We begin by studying the equilibrium behavior of the economy in the steady-state.

3.4.1 Steady-State Analysis:

In the steady-state, all real variables are stationary, with $\frac{Y_{t+1}}{Y_t}=\frac{K_{t+1}}{K_t}=\frac{\tilde{m}_{t+1}}{\tilde{m}_t}=\frac{w_{t+1}}{w_t}=1$. By imposing steady-state on (18), the rate of money growth pins down the inflation rate, with $\frac{P_{t+1}}{P_t}=\sigma$. Moreover, from (17) and (19), the following two loci characterize the steady-state equilibrium behavior of the economy.

$$K^{1-\rho-\alpha}=\frac{\sigma\alpha A}{I} \tag{20}$$

and

$$K^{1-\alpha-\rho}=\left(1-\frac{1}{\frac{1-\pi}{\pi}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}}I^{\frac{1-\theta}{\theta}}\sigma+1}\right)(1-\alpha)A \tag{21}$$

Equation (20) is the demand for capital, which is strictly decreasing in the nominal return from capital. By comparison, (21) is the supply of capital by the banking sector. It is clear from (21) that banks supply more capital as its return increases. Upon combining both loci, the nominal return that clears the capital market is the solution to the following polynomial:

$$\Gamma(I)\equiv\left(\sigma+\left[\frac{1-\pi}{\pi}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}}I^{\frac{1-\theta}{\theta}}\right]^{-1}\right)\frac{\alpha}{(1-\alpha)I}=1 \tag{22}$$

where $\Gamma(I)$ is such that: $\Gamma'(I)<0$, $\underset{I\to\infty}{lim}\,\Gamma\to 0$, and $\Gamma(\underline{I})=\left(\sigma\left[1-\frac{(1-\alpha-\rho)}{N}\right]+\frac{\pi}{1-\pi}\right)\frac{\alpha}{(1-\alpha)}$.

We proceed to establish existence and uniqueness of steady-state equilibria in the following proposition.

Proposition 1. *Existence and Uniqueness. Suppose* $\sigma\geq\underline{\sigma}$ *where* $\underline{\sigma}=\frac{\frac{1}{\alpha}-\frac{1}{1-\pi}}{1-\frac{(1-\alpha-\rho)}{N}}$. *Under this condition, a banking equilibrium where both money and capital are held exists and is unique.*

A simple examination of (22) indicates that the polynomial has a unique solution, I^*. From our discussion of the self-selection constraint, (13), a banking equilibrium exists if the equilibrium nominal return to capital (I^*) is above the threshold level, $\underline{I} > 1$ to prevent depositors from lying about their types ex-post. This takes place if the inflation rate is above some threshold level, $\underline{\sigma}$. Under this condition, $\Gamma\left(\underline{I}\right) > 1$ and $I^* > \underline{I}$. Moreover, the parameter space for existence is larger if the banking sector is more competitive. There is also greater support for existence if the externalities from capital formation are stronger.

We proceed to examine how the degree of banking competition affects the economy in the following Proposition.

Proposition 2. *Effects of Banking Competition.* $\frac{dK}{dN} > 0$, $\frac{dI}{dN} < 0$, $\frac{d\gamma}{dN} < 0$, *and* $\frac{d\frac{r^n}{r^m}}{dN} > 0$.

The intuition behind Proposition 2 is as follows. As the banking system becomes less competitive, banks allocate fewer resources towards capital formation and hold more liquid portfolios. This results in a lower supply of capital goods and a higher return. Moreover, given that banks hold more cash when the banking system is more concentrated, they provide their depositors with better insurance against liquidity shocks.

In the following Proposition, we discuss the effects of monetary policy.

Proposition 3. *Effects of Monetary Policy.* $\frac{dK}{d\sigma} > 0$, $\frac{dI}{d\sigma} > 0$, *and* $\frac{d\frac{r^n}{r^m}}{d\sigma} > 0$.

In this environment, a change in the rate of money creation has two primary effects on the real economy. First, banks hold a less liquid portfolio when the value of money is lower due to a higher opportunity cost. As more resources are allocated towards real investment, capital formation increases. Second, young workers receive larger injections from the government when the rate of money growth increases. This translates into higher young age income and deposits, which allows banks to raise their investment in different assets in the economy. Overall, a higher rate of money creation promotes capital formation. Furthermore, as money grows faster, the nominal return to capital is higher. Finally, given that banks hold less cash and the real return to money is lower when money grows faster, depositors receive less risk sharing.

Proposition 4. *Efficacy of Monetary Policy and Banking Competition.* $\frac{d\left(\frac{dK}{d\sigma}\right)}{dN} \geq (<)\, 0$ *if* $N \leq (>)\, \hat{N}(\rho)$, *where* $\hat{N}(\rho)$ *is defined in the appendix. Moreover,* $\frac{d\hat{N}}{d\rho} < 0$.

The result in Proposition 4 suggests that the marginal effects of monetary policy on capital formation depend on the degree of banking competition. However, the relationship is non-trivial. In particular, if the banking system is initially highly competitive, some consolidation in the banking system renders monetary policy more effective in stimulating investment. This takes place because banks will attempt to distort capital markets further and will hold more money balances as the sector consolidates. Consequently, the asset substitution channel of monetary policy is more effective. However, if the banking system is initially not too competitive, further consolidation weakens the ability of monetary policy to stimulate the economy because banks become so large that they realize that they have a significant on marginal revenue.

Moreover, the critical value at which the role of concentration changes depends on the extent of external economies from capital formation. As the externalities are stronger, the value of $\hat{N}$ decreases, implying that it is more likely that the efficacy of monetary policy improves as the sector is more concentrated. That is, the scope for the asset substitution channel of policy to be effective will be larger as external economies play a stronger role in economic activity. We turn to such implications immediately below.

The following Proposition summarizes the impact of the externalities for the level of development and risk sharing, two important factors in the level of welfare in the economy:

Proposition 5. *Effects of the Extent of Investment Externality. Suppose $A > A_0$, where A_0 is defined in the appendix. Under this condition, $\frac{dK}{d\rho} > 0$, $\frac{dI}{d\rho} < 0$, and $\frac{d\frac{r^n}{r^m}}{d\rho} > 0$.*

Intuitively, when the externality from capital investment gets stronger, the marginal return from capital is higher. This encourages banks to raise capital investment and hold less money. As the capital stock increases, its average return falls. Moreover, given that banks hold a less liquid portfolio, they provide less insurance against idiosyncratic liquidity shocks.

In our framework, there are two sources of market imperfections which play a role in the level of economic development. The first is due to the strategic behavior of imperfectly competitive intermediaries in the market for capital. Proposition 2 shows that higher degrees of concentration lead to lower capital accumulation, but greater risk-sharing. The second is due to the externality from capital formation. We turn now to the implications of these dual externalities and the role that they play in determining the ability of monetary policy to provide an environment that promotes economic development.

To do so, we proceed using various numerical examples based on the following parameter set: $\alpha = 0.33$, $A = 10$, $\pi = 0.5$ and $\theta = 0.9$. We begin by looking at activity in a banking sector which is highly concentrated, $N = 20$. As can be observed in the Table, the efficacy of monetary policy increases as external economies from equipment are stronger:

	dK/dσ		
	ρ=0	ρ=0.1	ρ=0.5
σ			
1.08	4.485	7.291	4136.099
1.09	4.455	7.246	4192.950
1.1	4.424	7.205	4249.387
1.11	4.394	7.164	4305.402
1.12	4.364	7.123	4360.984
1.13	4.334	7.083	4416.125
1.14	4.305	7.043	4470.809
1.15	4.275	7.003	4511.995

Table 1: The Effects of Monetary Policy in a Concentrated Banking Sector

However, the influence of policy is even more apparent in a setting with large external economies and a higher degree of competition. The following Table shows the same numbers except where $N = 100$.

	dK/dσ		
	ρ=0	ρ=0.1	ρ=0.5
σ			
1.08	4.514	7.356	4205.969
1.09	4.485	7.309	4264.066
1.1	4.453	7.267	4320.961
1.11	4.422	7.225	4377.416
1.12	4.391	7.183	4433.423
1.13	4.361	7.141	4488.974
1.14	4.330	7.099	4544.060
1.15	4.300	7.058	4598.674

Table 2: The Effects of Monetary Policy in a Less Concentrated Banking Sector

Therefore, in economies where both externalities are strong, the externalities conflict with each other. The positive external effects from equipment investment are weighed down by the negative implications from banking concentration. Monetary policy can play some role in promoting activity but the efficacy of monetary policy will be weaker. Consequently, increased concentration interrupts the ability of central banks to promote capital formation. *But, the mechanism is more effective if the externalities from capital formation are strong.* And, an abundance of research relies on such externalities as an important component of the development process.

4 Welfare Analysis

We proceed to study how monetary policy and the industrial organization of the banking system affect economic welfare. Following previous work such as Williamson (1986) and Ghossoub and Reed (2010), we use the steady-state expected utility of a typical generation of depositors as a proxy for welfare.

As we demonstrate in the appendix, the expected utility of a typical depositor in the steady-state can be expressed as:

$$u = \frac{[\alpha A]^{1-\theta}}{(1-\theta)(1-\pi)^{1-\theta}} \left(\frac{\pi}{(\alpha A)^{\frac{1-\theta}{\theta}} \sigma^{\frac{1-\theta}{\theta}}} \frac{K^{\frac{[1-(\rho+\alpha)(1-\theta)](1-\theta)}{\theta}}}{\left[1 - \frac{(1-\alpha-\rho)}{N}\right]^{\frac{1-\theta}{\theta}}} + (1-\pi) K^{(\rho+\alpha)(1-\theta)} \right) \tag{23}$$

We begin by examining the linkages between monetary policy and welfare. In particular, we assume that the monetary authority sets its inflation target, σ^*, in order to maximize (23). The properties of the welfare-maximizing money growth rate are as follows:

Proposition 6. *Suppose $\rho < \hat{\rho}$, where $\hat{\rho} = 1 - \alpha \left(2 + \frac{\pi}{1-\pi}\right) < 1 - \alpha$. Under this condition, $\frac{du}{d\sigma} < 0$ and $\sigma^* = \underline{\sigma}$. By comparison, suppose $\rho \in (\hat{\rho}, 1-\alpha)$. Under this condition, $\frac{du}{d\sigma} \geq (<)\, 0$ for all $\sigma \leq (>)\, \tilde{\sigma}$. Therefore, $\sigma^* = \tilde{\sigma}$. Finally, under both cases, we have $\frac{d\sigma^*}{dN} < 0$.*

From our result in Proposition 3, an increase in the rate of money creation involves a trade-off between higher capital formation and less risk sharing. When the external effects from capital investment are small, the marginal effects of a change in the inflation rate on capital formation are small as discussed in the previous section. Therefore, it is optimal for policymakers to focus on risk sharing rather than attempt to stimulate the economy. In comparison, when the externality is strong, lowering the value of money can be welfare improving as the gains in capital formation that result from higher inflation rates outweigh the loss in risk sharing. Finally, given that more concentration distorts the functioning of the banking system, it is optimal to reduce these distortions by lowering the value of money in order to promote investment.

Furthermore, we can show how the optimal rate of money growth varies with the external economies from investment and the degree of competition in the banking sector. Using the same set of parameters in Tables 1 and 2, we observe:

	ρ	0	0.005	0.01	0.0105	0.02	0.0205	0.03	0.0305	0.04	0.0405	0.05
N=20	σ*	1.066	1.066	1.065	1.068	1.112	1.114	1.159	1.162	1.209	1.211	1.259
N=100	σ*	1.037	1.037	1.037	1.039	1.083	1.085	1.130	1.132	1.178	1.180	1.228

Table 3: Optimal Monetary Policy, Strategic Behavior, and Investment Externalities

First, at low values of ρ, the optimal rate of money growth is the lowest in which a steady-state where money is dominated in rate of return exists. However, even in such economies, the optimal money growth rate is higher if the sector is more concentrated. As long as the external economies are somewhat strong, an increase in the positive external effects is associated with a higher optimal money growth rate. The absolute increases in the optimal rate are somewhat higher if the sector is more competitive suggesting that the optimal degree of intervention is conditional on both the degree of concentration and the extent of the external effects from investment.

Proposition 7. *Suppose $\rho < \hat{\rho}$. Under this condition, $\frac{du}{dN} < 0$. By comparison, suppose $\rho \in (\hat{\rho}, 1-\alpha)$ Under this condition, $\frac{du}{dN} \geq (<) 0$ if $N \leq (>) \tilde{N}$.*

If the external economies from production are weak, the optimal size distribution is to have a highly concentrated banking sector so that institutions will mostly focus on providing risk-sharing. By comparison, as the external economies are stronger, it becomes more beneficial to promote capital accumulation. Hence, the optimal size distribution balances the trade-offs between risk-sharing and encouraging capital formation.

To motivate the results in Proposition 7, we consider the following numerical example for different values of the rate of money growth, $\sigma = 1.05$ and $\sigma = 1.15$:

	ρ	0	0.0005	0.01	0.0105	0.02	0.0205	0.03	0.0305	0.04	0.0405	0.05
σ=1.05	N*	35.715	35.437	35.179	39.651	5491.006	22629.053	57504.472	58245.359	80924.099	135424.706	137845.416
σ=1.15	N*	6.436	6.431	6.341	6.457	10.056	10.370	26.625	29.063	1798.441	12209.256	14411.063

Table 4: Optimal Degree of Banking Competition

To begin, as indicated by the Proposition, the optimal banking system is a highly concentrated system when there is little positive external effect from the capital stock. However, as the externalities increase (and the literature stresses they are important in explaining the development process), the sector becomes much more competitive. We view these results to be quite informative about the consequences of consolidation that have occurred. For example, the Introduction highlights that there were nearly 19,000 different financial institutions in the United States in 1989 but the number dropped to about half a decade later. If the externalities from capital formation are somewhat strong, the results in Table 4 suggest that the concentration was likely to have had a strong impact on economic activity. However, in economies with higher inflation, the emphasis on risk-sharing in concentrated banking sectors can be socially valuable. Hence, the evidence in Table 4 indicates that more concentrated sectors are optimal in the presence of high rates of money growth. Yet, the benefits of concentration diminish in the presence of stronger external economies.

5 Banking Competition, Economic Growth, and Monetary Policy

In this section, we modify the environment studied above in a way that permits us to study the linkages between banking competition, economic growth, and monetary policy. In a one sector version of the Romer 'AK' model, the return in the capital market would be independent of the number of financial intermediaries and therefore the growth rate in the economy along the balanced growth path would not depend on concentration. In order to avoid this issue, we modify the one sector version to include two different sectors. Alternatively, the two different sectors could be viewed as two different regions which produce the same good, yet there are positive spillovers that occur across regions within the same island. That is, some benefits from external economies can flow across regions within the same island, but limited communication across islands still applies. In this manner, as we describe below, the region-specific externalities lead to downward-sloping demand curves for capital and allow for strategic behavior by Cournot-competing financial intermediaries in the capital market in each region.

Each island is divided into two regions, indexed by i, with $i = 1, 2$. At the beginning of each time period, a unit mass of young agents is born, with one half of the population residing in each region. Moreover, in each region, there are $N/2$ bankers, with a total population of bankers of N. In each region, there is a representative firm that uses capital and labor to produce the homogeneous consumption good. More specifically, we assume the factor markets are segregated. That is, a firm in region i can only use labor and capital available in its own region. Finally, capital and labor are completely immobile between regions.

Denote $K_{i,t}$ and $L_{i,t}$ to be the total amount of capital and labor available in region i at period t. The production technology of a representative firm in region i is of the form $Y_{i,t} = AK_t^{1-\alpha}K_{i,t}^{\alpha}L_{i,t}^{1-\alpha}$, where $K_t = K_{1,t} + K_{2,t}$ is the aggregate stock of capital across each region in the economy. As in Romer (1986), the externality from physical capital leads to perpetual growth. The remaining structure of the model is analogous to that in the previous section. We proceed to discuss the problem facing each agent in a particular region.

The representative firm in each region behaves competitively. Therefore, factors are paid their marginal products:

$$r_{i,t} = \alpha AK_t^{1-\alpha}K_{i,t}^{\alpha-1}L_{i,t}^{1-\alpha} \tag{24}$$

and

$$w_{i,t} = (1-\alpha) AK_t^{1-\alpha} K_{i,t}^{\alpha} L_{i,t}^{-\alpha} \tag{25}$$

5.1 A Typical Bank's Problem in Each Region

In this economy, banks in region i provide financial services to their clients residing in the same region. The total amount of deposits in each region is $\frac{1}{2}(w_{i,t} + \tau_t)$ where as in previous sections τ_t is the amount of real goods received by each young worker from the government. Given that banks provide similar financial services, deposits will be split evenly across banks in the same region. That is, each bank attracts $\frac{1}{N/2}\frac{1}{2}(w_{1,t} + \tau_t)$ in deposits. Furthermore, define $k_{i,t+1}$ and $m_{i,t}$ to be the amount of capital and real money balances held by each financial institution between periods t and $t+1$.

A typical bank's balance sheet is such that:

$$\frac{1}{N}(w_{i,t} + \tau_t) = m_{i,t} + k_{i,t+1} \tag{26}$$

In order to fulfill payments to movers and non-movers, the following resource constraints need to hold:

$$\pi \frac{1}{N} r_{i,t}^m (w_{i,t} + \tau_t) = m_{i,t} \frac{P_t}{P_{t+1}} \tag{27}$$

and

$$(1-\pi) \frac{1}{N} r_{i,t}^n (w_{i,t} + \tau_t) = r_{i,t+1} k_{i,t+1} \tag{28}$$

Furthermore, Cournot competition in the market for capital implies that banks face the inverse demand for capital in their region. That is:

$$r_{i,t+1} = \alpha A \left(\sum_{i=1}^{2} K_{i,t+1} \right)^{1-\alpha} K_{i,t+1}^{\alpha-1} L_{i,t+1}^{1-\alpha} \tag{29}$$

with $K_{i,t+1} = \sum_{j=1}^{N/2} k_{i,t+1}$.

If there were only firms in one region, the inverse demand for capital would be invariant to the number of intermediaries. However, there are two different regions with their own region-specific aggregate capital stocks. While firms in the region they are located take into account how they affect the capital stock in their own region, they take the aggregate stock for the other region as given since they do not operate in that location and capital and labor are immobile across regions. Hence, as is clear from (29), intermediaries in each region face downward-sloping demand curves for capital.

Finally, the following self-selection constraint must hold:

$$r_{i,t}^m \leq r_{i,t}^n \tag{30}$$

In sum, a typical bank in each region makes its portfolio choice and pricing to maximize the expected utility of its depositors subject to the above constraints. In particular, the bank solves the following problem:

$$\underset{r_{i,t}^{m},r_{i,t}^{n},m_{i,t},k_{i,t+1}}{Max} \frac{\pi\left[r_{i,t}^{m}\left(w_{i,t}+\tau_{t}\right)\right]^{1-\theta}+\left(1-\pi\right)\left[r_{i,t}^{n}\left(w_{i,t}+\tau_{t}\right)\right]^{1-\theta}}{1-\theta} \tag{31}$$

subject to (26) – (30).

The solution to the problem yields the demand for real money balances by one financial institution:

$$m_{i,t}=\frac{\frac{(w_{i,t}+\tau_{t})}{N}}{1+\frac{1-\pi}{\pi}\left[1-\frac{1-\alpha}{N}\right]^{\frac{1}{\theta}}I_{i,t}^{\frac{1-\theta}{\theta}}} \tag{32}$$

where $I_{i,t}=r_{i,t+1}\frac{P_{t+1}}{P_{t}}$. In addition, the relative return to depositors is such that:

$$\frac{r_{i,t}^{n}}{r_{i,t}^{m}}=\left[1-\frac{1-\alpha}{N}\right]^{\frac{1}{\theta}}I_{i,t}^{\frac{1}{\theta}} \tag{33}$$

5.2 General Equilibrium

We proceed to study symmetric equilibria. To begin, in equilibrium, all markets will clear. In particular, labor receives its marginal product, (25), with $w_{it}=w_{t}$ and the labor market clears, with $L_{i,t}=\frac{1}{2}$ and $\sum_{i=1}^{2}L_{i,t}=1$. Furthermore, by symmetry and the fact that the capital markets clear, we have $K_{t}=2K_{i,t}$, where $K_{i,t}=\frac{N}{2}k_{i,t}$. Using this information on (24) and (25), the expressions for wages and the real return to capital can be respectively expressed as:

$$w_{t}=\left(1-\alpha\right)AK_{t} \tag{34}$$

$$r_{t+1}=\alpha A \tag{35}$$

Even though intermediaries in each region face downward-sloping demand curves, *after imposing symmetry across the regional aggregates*, the equilibrium rental rate is independent of the number of firms. However, as we explain below, the growth rate along the balanced growth path will still depend on the degree of concentration in the banking sector because imperfectly competitive banks will distort the level of capital accumulation through excessive holdings of money balances.

Upon using a bank's balance sheet from (26) and the expression for transfers, the balance sheet constraint of a representative bank is:

$$\frac{1}{N}\left(w_{t}+\tau_{t}\right)=m_{t}+k_{t+1}$$

We next substitute for seigniorage transfers:

$$w_t + \frac{\sigma - 1}{\sigma} N m_t = N m_t + K_{t+1}$$

Solving for K_{t+1} :

$$K_{t+1} = \left(w_t - \frac{1}{\sigma} N m_t \right)$$

Based upon the amount of money balances held by a representative bank in equation (32):

$$K_{t+1} = \left(1 - \frac{1}{\sigma} \frac{1}{\frac{1-\pi}{\pi} \left[1 - (1-\alpha) \frac{1}{N} \right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}} + \frac{1}{\sigma}} \right) w_t$$

Therefore, it is clear that the law of motion for the aggregate capital stock depends on the degree of concentration because more concentrated banks hold more money balances:

$$K_{t+1} = \left(1 - \frac{1}{\sigma} \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]^{\frac{1}{\theta}} I_t^{\frac{1-\theta}{\theta}} + \frac{1}{\sigma}} \right) (1-\alpha) A K_t \tag{36}$$

In addition, from the evolution of money balances, (1), the inflation rate between t and $t+1$ is:

$$\frac{P_{t+1}}{P_t} = \sigma \frac{\tilde{m}_t}{\tilde{m}_{t+1}} \tag{37}$$

where $\tilde{m}_t = N m_{i,t}$. Furthermore, using the definition of I_t, (32), (34), (35), and (36) into (37), equilibrium in the money market requires that the nominal return to capital evolves according to:

$$I_{t+1}^{\frac{1-\theta}{\theta}} = \left(\frac{1-\alpha}{\alpha} I_t^{\frac{1}{\theta}} - \frac{1}{\frac{1-\pi}{\pi} \left[1 - \frac{1-\alpha}{N} \right]^{\frac{1}{\theta}}} \right) \frac{1}{\sigma} \tag{38}$$

In this manner, the system of equations, $(36) - (38)$, dictates the behavior of the economy over time. As we demonstrate in the appendix, Y, K, $\tilde{m}$, and w grow at the same rate g on the balanced growth path such that $\frac{Y_{t+1}}{Y_t} = \frac{K_{t+1}}{K_t} = \frac{\tilde{m}_{t+1}}{\tilde{m}_t} = \frac{w_{t+1}}{w_t} = g$. Moreover, I is stationary along the growth path. Substituting this information into the evolution equation of capital, the stationary growth rate of the economy is a solution to the following polynomial:

$$\Psi(g) = g + \frac{g^{\frac{1}{\theta}}}{\frac{1-\pi}{\pi}\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1}{\theta}} (\alpha A)^{\frac{1-\theta}{\theta}} \sigma^{\frac{1}{\theta}}} = (1-\alpha) A \tag{39}$$

and the nominal return to capital can be expressed as:

$$I = \alpha A \frac{\sigma}{g} \tag{40}$$

where $\frac{P_{t+1}}{P_t} = \frac{\sigma}{g}$ by (37) and the fact that real money balances grow at rate g on the balanced growth path.

We proceed to establish existence and uniqueness of a symmetric balanced growth path in the following proposition.

Proposition 8. *Existence and Uniqueness. Suppose* $\sigma \geq \underline{\sigma}_1$*, with* $\underline{\sigma}_1 = \frac{\frac{1}{\alpha} - \frac{1}{1-\pi}}{1 - \frac{(1-\alpha)}{N}}$*. Under this condition, a symmetric balanced growth path where both money and capital are held exists and is unique.*

A simple examination of (39) indicates that the polynomial has a unique non-negative solution, g^*. From the self-selection constraint, (33), the balanced growth path exists if the nominal return to capital, I^* is above a threshold level, $\underline{I}_1 = \frac{1}{1 - \frac{(1-\alpha)}{N}} > 1$ to prevent depositors from lying about their types ex-post. This requires the rate of money creation to be above some threshold level, $\underline{\sigma}_1$.

We proceed to examine how the degree of banking competition affects the economy in the following Proposition.

Proposition 9. $\frac{dg}{dN} > 0$, $\frac{d\frac{P_{t+1}}{P_t}}{dN} < 0$, $\frac{dI}{dN} < 0$, and $\frac{d\frac{r_t^n}{r_t^m}}{dN} > 0$.

The intuition behind Proposition 9 is analogous to Proposition 2. As the banking system becomes less competitive, banks allocate fewer resources towards capital formation and hold more liquid portfolios. This results in lower rates of economic growth. Therefore, policymakers should be concerned about the consolidation patterns that have developed over time in advanced economies.

In contrast to the economy in the absence of growth, the inflation rate is an endogenous outcome that is not completely pinned down by the rate of money growth. That is, the inflation rate responds to the growth rate of real money balances. Notably, economies with highly concentrated banking sectors will also exhibit higher inflation rates. In this manner, the degree of concentration has important inflationary consequences. As central banks have come to adopt a greater focus on price stability, our framework suggests that the ability to do so significantly depends on the degree of concentration. Nevertheless, given that banks hold more cash when the banking system is more concentrated, institutions in highly concentrated systems provide their depositors with better insurance against liquidity shocks.

In the following Proposition, we discuss the effects of monetary policy.

Proposition 10. $\frac{dg}{d\sigma} > 0$, $\frac{d\frac{P_{t+1}}{P_t}}{d\sigma} > 0$, $\frac{dI}{d\sigma} > 0$, *and* $\frac{d\frac{r_t^n}{r_t^m}}{d\sigma} > 0$.

Thus, monetary policy can still play an important role in providing a favorable growth environment. A higher rate of money growth is associated with a Tobin asset substitution channel which promotes economic growth. And, policy produces other standard economic outcomes – higher inflation rates and nominal returns. However, as we explain, the efficacy of monetary policy depends on the degree of concentration in non-trivial ways:

Proposition 11. *Efficacy of Monetary Policy and Banking Competition.* $\frac{\frac{dg}{d\sigma}}{dN} \geq (<)\, 0$ *if* $N \leq (>)\, \hat{N}$, *where* $\hat{N}$ *is defined in the appendix. Moreover,* $\frac{\frac{d\frac{P_{t+1}}{P_t}}{d\sigma}}{dN} > 0$.

The result in Proposition 4 suggests that the marginal effects of monetary policy on economic growth depend on the degree of banking competition. However, the relationship is non-trivial. In particular, if the banking system is initially highly competitive, some consolidation in the banking system renders monetary policy more effective in stimulating economic growth. However, if the banking system is initially not too competitive, further consolidation weakens the ability of monetary policy to stimulate growth.

Interestingly, as the banking system becomes more concentrated, the ability of monetary policy to create inflation is unambiguously hampered. This prediction from our framework provides some insights as to why central banks in the developed world have been struggling to re-inflate their economies over time.

6 Conclusion

Since the global financial crisis, there has been a significant amount of concern about the presence of large-scale financial intermediaries which affects the competitive landscape of the banking sector in advanced economies. In light of this issue, this paper develops a framework to demonstrate how the degree of concentration impacts capital accumulation and economic growth. As is standard in the growth literature, we incorporate production externalities from the aggregate capital stock which promote economic development and growth over time. Notably, the model demonstrates that higher degrees of concentration distort economic activity by interrupting the externalities from capital investment. We also show that the ability of monetary policy to provide a favorable growth environment and achieve price stability is hampered by higher degrees of concentration. Consequently, the task of central banking will be more difficult as the sector further consolidates over time. Furthermore, the developing world will not be immune to these challenges facing regulators and policymakers in advanced economies.

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7 Technical Appendix

1. **Proof of Proposition 2.** To begin, we use some simple algebra to re-write the capital market clearing condition in the following manner. First, from (6) and (11), the demand for real money in the steady-state can be expressed as:

$$m = \frac{k}{\frac{1-\pi}{\pi}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}} I^{\frac{1-\theta}{\theta}}} \tag{41}$$

Moreover, using (15), (16), the definition of I, and the fact that $K = Nk$ into (41), we get:

$$\frac{Nm}{\sigma w} = \frac{1}{\frac{1-\alpha}{\alpha}\frac{1-\pi}{\pi}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}} I^{\frac{1}{\theta}}} \tag{42}$$

Finally, from (2), (6), and (42), the aggregate supply of capital by the banking sector is such that:

$$\frac{K^{1-\alpha-\rho}}{(1-\alpha)A} = 1 - \frac{\pi}{1-\pi}\frac{\alpha}{1-\alpha}\frac{1}{\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}} I^{\frac{1}{\theta}}} \tag{43}$$

where $I = \sigma\alpha A K^{\rho+\alpha-1}$. Thus, the aggregate stock of capital is a solution to the following polynomial:

$$\frac{K^{1-\alpha-\rho}}{(1-\alpha)A} = 1 - \frac{1}{\frac{1-\alpha}{\alpha}\frac{1-\pi}{\pi}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}} (\alpha A)^{\frac{1}{\theta}}}\frac{K^{\frac{1-\rho-\alpha}{\theta}}}{\sigma^{\frac{1}{\theta}}} \tag{44}$$

With some algebra, the total differentiation of (44) with respect to N yields:

$$\frac{N}{K}\frac{dK}{dN} = \frac{\frac{\frac{(1-\alpha-\rho)}{N}}{1-\frac{(1-\alpha-\rho)}{N}}}{\frac{(1-\alpha-\rho)\theta K^{1-\alpha-\rho}}{(1-\alpha)A\left(1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}\right)} + (1-\rho-\alpha)} > 0 \tag{45}$$

By diminishing returns, $\frac{dr}{dN} < 0$ and $\frac{dI}{dN} < 0$. We proceed to show the effect of N on γ and $\frac{r^n}{r^m}$. By definition of γ and from the bank's balance sheet, (6),

$$\gamma = \frac{1}{\frac{1}{\sigma}\frac{1}{1-\frac{K}{w}} + \frac{\sigma-1}{\sigma}} \tag{46}$$

which obviously suggests that each bank allocates smaller resources towards cash reserves when the banking sector is more competitive as $\frac{K}{w} = \frac{K^{1-\alpha-\rho}}{(1-\alpha)A}$ is increasing in N. As for the effects on risk sharing, using (6) and (42) into (13), the relative return to depositors can be expressed as:

$$\frac{r^n}{r^m} = \frac{\pi}{1-\pi}\frac{\alpha}{1-\alpha}\frac{1}{\left(1-\frac{K}{w(K)}\right)}$$

Therefore, higher N leads to higher K and less risk sharing. This completes the proof of Proposition 2.

2. **Proof of Proposition 3.** Differentiating (44) with respect to σ and some simplification yields:

$$\frac{\sigma}{K}\frac{dK}{d\sigma} = \frac{1}{\theta(1-\alpha-\rho)}\frac{1}{\frac{1}{1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}}+\frac{1-\theta}{\theta}} > 0 \tag{47}$$

Moreover, from the polynomial yielding I, (22), it is trivial to show that $\frac{dI}{d\sigma} > 0$. The impact of σ on γ and $\frac{r^n}{r^m}$ directly follows. This completes the proof of Proposition 3.

3. **Proof of Proposition 4.** From (47), define $z(K) = \frac{1}{1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}}$ and differentiating (47) with respect to N to get:

$$\theta(1-\alpha-\rho)\sigma\frac{dK}{d\sigma dN} = \frac{\left[\left[z(K)+\frac{1-\theta}{\theta}\right]-z'(K)K\right]\frac{dK}{dN}}{\left[z(K)+\frac{1-\theta}{\theta}\right]^2}$$

Given that $\frac{dK}{dN} > 0$, then $\frac{dK}{d\sigma dN} \geq 0$ if $\left[\left[z(K)+\frac{1-\theta}{\theta}\right]-z'(K)K\right] \geq 0$. Using the definition of $z(K)$, this condition holds if:

$$1+\frac{1-\theta}{\theta}\left(1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}\right) \geq \frac{\frac{K^{1-\alpha-\rho}}{A}}{1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}} \tag{48}$$

It is easy to verify that the term on the left-hand-side of (48) is decreasing in N while the term on the right-hand-side is increasing in N. Therefore, there exists an $\hat{N}$ such that (48) holds with equality. The result in the Proposition directly follows.

Finally, we demonstrate that $\frac{d\hat{N}}{d\rho} < 0$. It suffices to show that $\frac{K}{w} = \frac{K^{1-\alpha-\rho}}{(1-\alpha)A}$ is increasing in ρ for a given value of N. From the expression for the relative return to depositors, (13) and the expression for the supply of capital, (43), (43) can be expressed as:

$$\frac{K^{1-\alpha-\rho}}{(1-\alpha)A} = 1-\frac{\pi}{1-\pi}\frac{\alpha}{1-\alpha}\frac{1}{\frac{r^n}{r^m}}$$

As we demonstrate in the proof of Proposition 5 that $\frac{d\frac{r^n}{r^m}}{d\rho} > 0$, this also implies that $\frac{d\left(\frac{K}{w}\right)}{d\rho} > 0$. In this manner, for a given value of N, the term on the left-hand-side (right-hand-side) of (48) is decreasing (rising) in ρ. This directly implies that $\frac{d\hat{N}}{d\rho} < 0$. This completes the proof of Proposition 4.

4. **Proof of Proposition 5.** We begin by differentiating (22) with respect to ρ and some simplification to get:

$$\frac{dI}{d\rho} = \frac{-\frac{1}{\theta}\frac{1}{N}\left[1 - \frac{(1-\alpha-\rho)}{N}\right]^{-1}}{\left(\frac{1-\theta}{\theta}I^{-1} + \frac{(1-\alpha)}{\alpha}\frac{1}{\frac{(1-\alpha)I}{\alpha}-\sigma}\right)} < 0 \tag{49}$$

Next, we know that $I = \sigma\alpha AK^{\rho+\alpha-1}$. Taking the log and differentiating with respect to ρ :

$$\ln K - \frac{1}{I}\frac{dI}{d\rho} = (1-\rho-\alpha)\frac{1}{K}\frac{dK}{d\rho} \tag{50}$$

we know that $\frac{dI}{d\rho} < 0$. Clearly, if $K > 1$, $\ln K > 0$ and $\frac{dK}{d\rho} > 0$. However, if $K < 1$, $\ln K < 0$ and $\frac{dK}{d\rho}$ can be negative. For this comparative static, we choose to focus on cases where $K > 1$. From the polynomial yielding K, (44), the term on the left-hand-side is increasing in K while that on the right-hand side is falling in K. Moreover, for a given K, as A rises, LHS falls while RHS rises. Overall, $\frac{dK}{dA} > 0$. $K^* > 1$ if at $K = 1$, $LHS < RHS$. This condition can be written as:

$$\frac{1}{(1-\alpha)A} + \frac{1}{\frac{1-\alpha}{\alpha}\frac{1-\pi}{\pi}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}}(\alpha A)^{\frac{1}{\theta}}}\frac{1}{\sigma^{\frac{1}{\theta}}} < 1$$

Therefore, there exists an A_0 such that this condition holds with equality. For $A > A_0$, $K > 1$ and $\frac{dK}{d\rho} > 0$. This necessarily happens because as ρ increases, the exponent on K drops, so the term on the LHS shifts down, while the term on the RHS shifts up. The term in the denominator also supports the upward shift of the term on the RHS.

We proceed by examining the effects of ρ on risk sharing. Taking the natural log of (13) and differentiating with respect to ρ, we get:

$$\frac{1}{\frac{r^n}{r^m}}\frac{d\frac{r^n}{r^m}}{d\rho} = \frac{1}{\theta}\frac{1}{[N-(1-\alpha-\rho)]} + \frac{1}{\theta}\frac{1}{I}\frac{dI}{d\rho}$$

Therefore, $\frac{d\frac{r^n}{r^m}}{d\rho} > 0$ if:

$$\frac{\rho}{[N-(1-\alpha-\rho)]} > -\frac{\rho}{I}\frac{dI}{d\rho} \tag{51}$$

Upon using (49) into (51) and some simplification, $\frac{d\frac{r^n}{r^m}}{d\rho} > 0$ if:

$$-\sigma < 0$$

which always holds. This completes the proof of Proposition 5.

5. **Proof of Proposition 6.** We begin by deriving the expression for welfare, (23). From a bank' s problem, we substitute the resource constraints,

(7) − (8) and the expression for the rental rate, (16) into the expected utility function, (5) to get:

$$u = \frac{\pi \left[\frac{N}{\pi}\frac{1}{\sigma}m\right]^{1-\theta} + (1-\pi)\left[\frac{1}{1-\pi}\alpha A K^{\rho+\alpha}\right]^{1-\theta}}{1-\theta}$$

Subsequently, using the expression for $\frac{Nm}{w}\frac{1}{\sigma}$ from our work above, (42) and the fact that $I = \sigma\alpha A K^{\rho+\alpha-1}$ to obtain the expression for welfare in the text, (23).

We proceed to differentiate (23) with respect to σ to get:

$$\frac{du}{d\sigma} = \frac{\frac{\pi\left(\frac{[1-(\rho+\alpha)(1-\theta)](1-\theta)}{\theta}\frac{1}{K}\frac{dK}{d\sigma} - \frac{1-\theta}{\theta}\frac{1}{\sigma}\right)}{\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1-\theta}{\theta}}(\alpha A)^{\frac{1-\theta}{\theta}}\sigma^{\frac{1-\theta}{\theta}}} K^{\frac{[1-(\rho+\alpha)(1-\theta)](1-\theta)}{\theta}} + (\rho+\alpha)(1-\theta)(1-\pi)K^{(\rho+\alpha)(1-\theta)-1}\frac{dK}{d\sigma}}{\frac{(1-\theta)(1-\pi)^{1-\theta}}{[\alpha A]^{1-\theta}}}$$

With some simplifying algebra, $\frac{du}{d\sigma} \geq 0$ if:

$$\frac{\sigma}{K}\frac{dK}{d\sigma} \geq \frac{1}{[1-(\rho+\alpha)(1-\theta)] + (\rho+\alpha)\theta\frac{1-\pi}{\pi}\left(\sigma^{\frac{1}{\theta}}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}}(\alpha A)^{\frac{1}{\theta}}K^{\left[\frac{-1+(\rho+\alpha)}{\theta}\right]}\right)^{1-\theta}} \tag{52}$$

From the polynomial yielding K, (44), we have:

$$\sigma^{\frac{1}{\theta}}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1}{\theta}}(\alpha A)^{\frac{1}{\theta}} = \frac{1}{\frac{1-\alpha}{\alpha}\frac{1-\pi}{\pi}}\frac{K^{\frac{1-\rho-\alpha}{\theta}}}{1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}}$$

Substitute into (52) so that with and some simplification, the condition becomes:

$$\frac{\sigma}{K}\frac{dK}{d\sigma} \geq \frac{1}{[1-(\rho+\alpha)(1-\theta)] + (\rho+\alpha)\theta\left(\frac{1-\pi}{\pi}\right)^{\theta}\left(\frac{\alpha}{1-\alpha}\frac{1}{1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}}\right)^{1-\theta}}$$

Finally, using the expression for $\frac{\sigma}{K}\frac{dK}{d\sigma}$ from (47), $\frac{du}{d\sigma} \geq 0$ if:

$$\psi(\sigma) \equiv \left(1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}\right) + (\rho+\alpha)\left(\frac{1-\pi}{\pi}\right)^{\theta}\left(\frac{\alpha}{1-\alpha}\right)^{1-\theta}\left(1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}\right)^{\theta} \geq (1-\alpha-\rho)$$

Given that $\frac{dK}{d\sigma} > 0$, $\psi'(\sigma) < 0$. Define $\tilde{\sigma} : \psi(\sigma) = (1-\alpha-\rho)$. For all $\sigma > \tilde{\sigma}$, $\frac{du}{d\sigma} < 0$ and for all $\sigma \leq \tilde{\sigma}$, $\frac{du}{d\sigma} \geq 0$. In addition, $\frac{\partial\psi}{\partial N} < 0$. Therefore, for a given σ, the locus shifts down. As a result $\frac{d\tilde{\sigma}}{dN} < 0$. Finally, we need to compare $\tilde{\sigma}$ to $\underline{\sigma}$. Recall the existence condition: $\sigma \geq \underline{\sigma} = \frac{\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}}{1-\frac{(1-\alpha-\rho)}{N}}$. Therefore, $\underline{\sigma} > \tilde{\sigma}$ if $\psi(\underline{\sigma}) < (1-\alpha-\rho)$. Upon using the definition of $\underline{\sigma}$, this condition reduces into $\rho < 1 - \left(2 + \frac{\pi}{1-\pi}\right)\alpha = \hat{\rho}$. Consequently, the result stated in Proposition 6 holds. This completes the proof of Proposition 6.

6. **Proof of Proposition 7.** Differentiating the welfare function, (23) with respect to N and some simplification yields:

$$\frac{du}{dN} = \frac{\frac{\pi\left(\frac{[1-(\rho+\alpha)(1-\theta)]}{\theta}\frac{dK}{dN} - \frac{(1-\alpha-\rho)}{N}\frac{1}{\theta}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{-1}\right)K^{\frac{[1-(\rho+\alpha)(1-\theta)](1-\theta)}{\theta}}}{(\alpha A)^{\frac{1-\theta}{\theta}}\sigma^{\frac{1-\theta}{\theta}}\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1-\theta}{\theta}}} + (\rho+\alpha)(1-\pi)K^{(\rho+\alpha)(1-\theta)-1}\frac{dK}{dN}}{\left(\frac{1-\pi}{\alpha A}\right)^{1-\theta}}$$

Next, with some algebra, $\frac{du}{dN} \geq 0$ if:

$$\frac{N}{K}\frac{dK}{dN} \geq \frac{\frac{\frac{(1-\alpha-\rho)}{N}}{1-\frac{(1-\alpha-\rho)}{N}}}{[1-(\rho+\alpha)(1-\theta)] + (\rho+\alpha)(\alpha A)^{\frac{1-\theta}{\theta}}\sigma^{\frac{1-\theta}{\theta}}\theta\left(\frac{1-\pi}{\pi}\right)\left[1-\frac{(1-\alpha-\rho)}{N}\right]^{\frac{1-\theta}{\theta}}K^{-\left[\frac{1-(\rho+\alpha)}{\theta}\right](1-\theta)}} \tag{53}$$

Upon using the the polynomial yielding K, (44), condition (53) can be written as:

$$\frac{N}{K}\frac{dK}{dN} \geq \frac{\frac{\frac{(1-\alpha-\rho)}{N}}{1-\frac{(1-\alpha-\rho)}{N}}}{[1-(\rho+\alpha)(1-\theta)] + (\rho+\alpha)\theta\left(\frac{1-\pi}{\pi}\right)^{\theta}\frac{\left(\frac{\alpha}{1-\alpha}\right)^{1-\theta}}{\left(1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}\right)^{1-\theta}}}$$

Furthermore, substitute for $\frac{N}{K}\frac{dK}{dN}$ from (47), then $\frac{du}{dN} \geq 0$ if:

$$1 + \left(\frac{1-\pi}{\pi}\right)^{\theta}\left(\frac{\alpha}{1-\alpha}\right)^{1-\theta}\left(1-\frac{K^{1-\alpha-\rho}}{(1-\alpha)A}\right)^{\theta} \geq \frac{K^{1-\alpha-\rho}}{(\rho+\alpha)(1-\alpha)A} \tag{54}$$

Therefore, for $N \leq \tilde{N}$, $\frac{du}{dN} \geq 0$. Moreover, $\frac{d\tilde{N}}{d\sigma} < 0$. Finally, from the existence condition, we need $\sigma \geq \frac{\frac{1-\alpha}{\alpha} - \frac{\pi}{1-\pi}}{1-\frac{(1-\alpha-\rho)}{N}}$, which can also be written as a condition on N. In particular, a banking equilibrium exists if: $N \geq \frac{(1-\alpha-\rho)}{1-\frac{\frac{1-\alpha}{\alpha}-\frac{\pi}{1-\pi}}{\sigma}} = \underline{N}$, where at $\underline{N}$ we have complete risk sharing. Evaluate the supply of capital, (21) at $\underline{N}$ to get:

$$1 - \frac{K^{1-\alpha-\rho}}{(1-\alpha)A} = \frac{1}{\frac{1-\alpha}{\alpha}\frac{1-\pi}{\pi}}$$

In this manner, $\underline{N} > \tilde{N}$ if (54) strictly holds when evaluated at $\underline{N}$. Using this information, $\underline{N} > \tilde{N}$ if $\rho < 1 - \alpha\left(\frac{2-\pi}{1-\pi}\right)$. The result in the Proposition directly follows. This completes the proof of Proposition 7.

The model with Endogenous Growth:

We begin by showing that along the balanced growth path, m and K grow at the same rate. Moreover, inflation and the nominal return to capital are constant. Define $\mu_{t+1} = \frac{P_{t+1}}{P_t}$ to be the inflation rate in $t+1$ and $g_{m,t+1}$ to be the growth rate in real money balances between t and $t+1$. Using this information in the evolution of real money balances, (1):

$$\mu_{t+1} = \sigma \frac{1}{g_{m,t+1}}$$

Updating one period and re-writing to get:

$$\frac{\mu_{t+2}}{\mu_{t+1}} = \frac{g_{m,t+1}}{g_{m,t+2}}$$

Along the balanced growth path, real money balances grow at the same rate over time therefore $\frac{\mu_{t+2}}{\mu_{t+1}} = 1$. Moreover, by the definition of I_t, $I_t = r_{t+1}\frac{P_{t+1}}{P_t} = \alpha A \mu_{t+1}$. Along the balanced growth path, we have $\frac{I_{t+1}}{I_t} = \frac{\mu_{t+2}}{\mu_{t+1}} = 1$. Finally, using the expression for transfers, (2), a bank's balance sheet, (26), the demand for money, (32) in region i can be written as:

$$\frac{N}{2} m_{i,t} = \frac{K_{i,t+1}}{\frac{1-\pi}{\pi}\left[1-(1-\alpha)\frac{1}{N}\right]^{\frac{1}{\theta}} I_{i,t}^{\frac{1-\theta}{\theta}}} \tag{55}$$

where $\tilde{m}_t = N m_{i,t}$ as discussed in the text. Therefore:

$$\frac{\tilde{m}_{t+1}}{\tilde{m}_t} = \frac{K_{t+2}}{K_{t+1}} \frac{I_t^{\frac{1-\theta}{\theta}}}{I_{t+1}^{\frac{1-\theta}{\theta}}}$$

along the balanced growth path, we just showed that $\frac{I_{t+1}}{I_t} = 1$. Therefore:

$$\frac{\tilde{m}_{t+1}}{\tilde{m}_t} = \frac{K_{t+2}}{K_{t+1}} = g$$

7. **Proof of Proposition 9.** Recall the polynomial that yields g from the text:

$$\Psi(g) = g + \frac{g^{\frac{1}{\theta}}}{\frac{1-\pi}{\pi}\left[1-\frac{(1-\alpha)}{N}\right]^{\frac{1}{\theta}} (\alpha A)^{\frac{1-\theta}{\theta}} \sigma^{\frac{1}{\theta}}} = (1-\alpha) A$$

Clearly $\Psi'(g) > 0$ and the system has a unique solution, g^*. It is trivial to show that $\frac{\partial \Psi}{\partial N} < 0$. Therefore, $\frac{dg^*}{dN} > 0$. Moreover, since $I = \alpha A \frac{\sigma}{g_k}$, it directly follows that $\frac{dI}{dN} < 0$. Furthermore, given that $\frac{P_{t+1}}{P_t} = \frac{\sigma}{g_k}$, $\frac{d\frac{P_{t+1}}{P_t}}{dN} < 0$.

We proceed to show the effects of banking competition on risk sharing. It is easy to verify that the bank's money demand equation derived above, (55) can be written as:

$$Nm_{i,t} = \frac{K_{t+1} I_t}{\frac{1-\pi}{\pi}\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1}{\theta}}} \tag{56}$$

Using the fact that $I_t = \alpha A \frac{\sigma}{g_k} = \alpha A \frac{\sigma K_t}{K_{t+1}}$, (56) becomes:

$$Nm_{i,t} = \frac{\alpha A \sigma K_t}{\frac{1-\pi}{\pi}\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1}{\theta}}} \tag{57}$$

Therefore:

$$\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1}{\theta}} = \frac{\alpha A K_t}{\frac{1-\pi}{\pi}\frac{Nm_{i,t}}{\sigma}} \tag{58}$$

From the bank's balance sheet, (26) and the expression for transfers, (2):

$$\sigma\left(w_t - K_{t+1}\right) = Nm_{i,t} \tag{59}$$

Using this information and the fact that $w_t = (1-\alpha) A K_t$, into (58) :

$$\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1}{\theta}} I_t^{\frac{1}{\theta}} = \frac{\alpha A}{\frac{1-\pi}{\pi}\left((1-\alpha) A - \frac{K_{t+1}}{K_t}\right)} \tag{60}$$

Finally, using (60) into (33) to get:

$$\frac{r_t^n}{r_t^m} = \frac{\alpha A}{\frac{1-\pi}{\pi}\left((1-\alpha) A - g\right)}$$

In this manner, banks provide less insurance against liquidity risk when the banking system becomes more competitive since $\frac{dg^*}{dN} > 0$. This completes the proof of Proposition 9.

8. **Proof of Proposition 10.** Differentiating the polynomial yielding g with respect to σ with some algebra we get:

$$\frac{dg}{d\sigma} = \frac{\sigma^{-1}}{\theta\frac{1-\pi}{\pi}\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1}{\theta}} (\alpha A)^{\frac{1-\theta}{\theta}} \sigma^{\frac{1}{\theta}} g^{-\frac{1}{\theta}} + g^{-1}} > 0 \tag{61}$$

The effects of σ on I can be observed by re-writing (39) using the fact that $I = \alpha A \frac{\sigma}{g_k}$ to obtain:

$$\sigma\frac{1}{I} + \frac{1}{\frac{1-\pi}{\pi}\left[1 - \frac{(1-\alpha)}{N}\right]^{\frac{1}{\theta}} I^{\frac{1}{\theta}}} = \frac{1-\alpha}{\alpha}$$

It is trivial to verify that $\frac{dI}{d\sigma} > 0$. Finally, along the balanced growth path, the inflation rate is given by $\frac{P_{t+1}}{P_t} = \frac{\sigma}{g_k}$. Differentiating with respect to σ and some algebra:

$$\frac{d\frac{P_{t+1}}{P_t}}{d\sigma} = \frac{\frac{1}{(1-\alpha)A}}{1 + \frac{1-\theta}{\theta}\left(1 - \frac{g}{(1-\alpha)A}\right)} > 0 \tag{62}$$

This completes the proof of Proposition 10.

9. **Proof of Proposition 11.** Using (39), it is easily verified that (61) can be expressed as:

$$\sigma\theta\frac{dg}{d\sigma} = \frac{g}{z(g) + \frac{1-\theta}{\theta}} \tag{63}$$

where $z(g) = \frac{1}{1-\frac{g}{(1-\alpha)A}}$. Differentiating (63) with respect to N yields:

$$\sigma\theta\frac{dg}{d\sigma dN} = \frac{\left(z(g) + \frac{1-\theta}{\theta} - gz'(g)\right)\frac{dg}{dN}}{\left[z(g) + \frac{1-\theta}{\theta}\right]^2}$$

Using the definition of z, $\frac{dg}{d\sigma dN} > 0$ if:

$$1 + \frac{1-\theta}{\theta}\left(1 - \frac{g_k}{(1-\alpha)A}\right) > \frac{1}{\left(\frac{(1-\alpha)A}{g_k} - 1\right)}$$

Given that g is increasing in N, there exists a value of N, $\hat{N}$ such that the above holds with equality. For all $N \leq (>)\,\hat{N}$, $\frac{dg}{d\sigma dN} \geq (<)\,0$. Finally, from the (62), it is trivial to see that $\frac{d\frac{P_{t+1}}{P_t}}{d\sigma dN} > 0$ since $\frac{dg}{dN} > 0$. This completes the proof of Proposition 11.